

**Mathematics Teacher Preparation in California:
Standards of Quality and Effectiveness for
Subject Matter Programs**

**Handbook for Teacher Educators
and Program Reviewers**

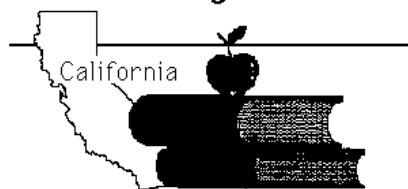
**Commission on Teacher Credentialing
State of California**

1992

Mathematics Teacher Preparation in California: Standards of Quality and Effectiveness for Subject Matter Programs

**Created and Recommended by the
Mathematics Teacher Preparation and
Assessment Advisory Panel
(1989-91)**

**Commission on Teacher
Credentialing**



**Adopted and Implemented by the
Commission on Teacher Credentialing
State of California
1812 Ninth Street
Sacramento, California
1992**

The Commission on Teacher Credentialing

State of California

Pete Wilson, Governor

1992

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**Commission on Teacher Credentialing
1989-91**

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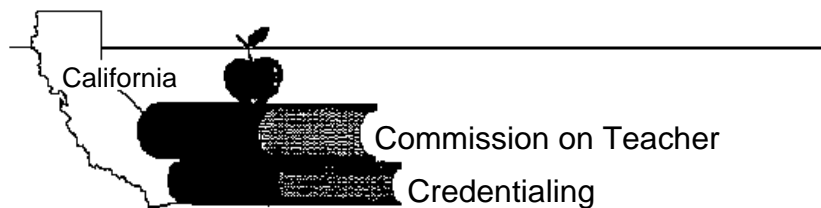
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Part 1

Introduction to

Mathematics Teaching Standards



Standards and Credentials for Teachers of Mathematics: Foreword by the Commission on Teacher Credentialing

One of the purposes of education is to enable students to learn the important subjects of the school curriculum, including mathematics. Each year in California, more than one million students enroll in mathematics classes with teachers who are certified by the Commission on Teacher Credentialing to teach those classes in public schools. The future well-being of California and the nation depends in part on how well these students learn to use mathematics thoughtfully and skillfully. Their ability to do so depends substantially on the quality of the teachers' preparation in mathematics, and in the teaching of mathematics.

The Commission is the agency of California government that certifies the competence of teachers and other professionals who serve in the public schools. As the policy-making body that establishes and maintains standards for the education profession in the State, the Commission is concerned about the quality and effectiveness of the preparation of teachers and other school practitioners. On behalf of the education profession and the general public, the Commission's most important responsibility is to establish and implement strong, effective standards of quality for the preparation and assessment of credential candidates.

In 1988 and 1992 the Legislature and the Governor enacted laws that strengthened the professional character of the Commission, and enhanced its authority to establish rigorous standards for the preparation and assessment of prospective teachers. As a result of these reform laws (Senate Bills 148 and 1422, Bergeson), a majority of the Commission members are professional educators, and the agency is responsible for establishing acceptable levels of quality in teacher preparation and acceptable levels of competence in beginning teachers. To implement the reform statutes, the Commission is developing new standards and other policies collaboratively with representatives of postsecondary institutions and statewide leaders of the education profession.

To ensure that future teachers of mathematics have the finest possible education, the Commission decided to establish a panel of experts to review recent developments in mathematics education, and to recommend new standards for the academic preparation of mathematics teachers in California. The Commission's Executive Director invited colleges, universities, professional organizations, school districts, county offices of education and other state agencies to nominate distinguished professionals to serve on this panel. After receiving nearly 100 nominations, the Executive Director appointed the Mathematics Teacher Preparation and Assessment Advisory Panel (see page ii). These seventeen professionals were selected for their expertise in mathematics teaching, their effectiveness as teachers and professors of mathematics, and their leadership in the field of mathematics education. The panel represented the diversity of California educators, and included mathematics teachers and curriculum specialists as well as university professors and administrators. The panel met on several occasions during 1989 and 1990 to discuss, draft and develop the standards in this handbook. The Commission is deeply grateful to the panelists for their conscientious work in addressing many complex issues related to excellence in the subject matter preparation of mathematics teachers.

The Mathematics Teaching Credential

The Single Subject Teaching Credential in Mathematics authorizes an individual to teach mathematics classes in departmentalized settings. The holders of this credential may teach at any grade level, but departmentalized teaching of mathematics usually occurs in grades seven through twelve. The Commission asked the Mathematics Teacher Preparation and Assessment Advisory Panel to recommend new policies to ensure that future teachers of mathematics are prepared to instruct the subjects that are most commonly taught in mathematics classes. In 1988-89, when the advisory panel was established, 50 percent of all mathematics classes in California public schools were general courses in basic or remedial mathematics for students in grades seven through twelve. The remaining 50 percent of the classes taught by mathematics teachers were more specialized courses in the following subjects.

Algebra and Beginning Algebra	18% of All Mathematics Classes
Computer Literacy and Programming	10%
Plane and Solid Geometry	9%
Pre-Calculus and Calculus	3%
Trigonometry	1%
Other Mathematics Subjects	8%

The standards and other policies in this document are designed to prepare teachers for basic and remedial classes in mathematics, as well as the more specialized courses listed above.

Subject Matter Preparation Programs for Prospective Teachers

An applicant for a Single Subject Teaching Credential must demonstrate subject matter competence in one of two ways. The applicant may earn a passing score on a subject matter examination that has been adopted by the Commission. Alternatively, the prospective teacher may complete a subject matter preparation program that has been approved by the Commission (Education Code Sections 44280 and 44310). Regionally accredited colleges and universities that wish to offer subject matter programs for prospective teachers must submit those programs to the Commission for approval.

In California, subject matter preparation programs for prospective teachers are not the same as undergraduate degree programs. Postsecondary institutions govern academic programs that lead to the award of degrees, including baccalaureate degrees in mathematics. The Commission sets standards for academic programs that lead to the issuance of credentials, including the Single Subject Teaching Credential in Mathematics. An applicant for a teaching credential must have earned a Bachelor's degree from an accredited institution, but the degree may be in a subject other than the one to appear on the credential. Similarly, degree programs for undergraduate students in mathematics may or may not fulfill the Commission's standards for subject matter preparation. Completing a subject matter program that satisfies the standards enables a candidate to qualify for the Single Subject Credential in Mathematics.

The Commission asked the Mathematics Teacher Preparation and Assessment Advisory Panel to create new standards of program quality and effectiveness that could be used to review and approve subject matter preparation programs. The Commission requested the development of standards that emphasize the knowledge, skills and perspectives that teachers must have in order to teach mathematics effectively in public schools.

Standards of Program Quality and Effectiveness

In recent years the Commission has thoroughly redesigned its policies regarding the preparation of education professionals and the review of preparation programs in colleges and universities. In initiating these reforms, the Commission embraced the following principles or premises regarding the governance of educator preparation programs. The Commission asked the Mathematics Teacher Preparation and Assessment Advisory Panel to apply these general principles to the creation of standards for subject matter programs in mathematics.

- (1) *The status of teacher preparation programs in colleges and universities should be determined on the basis of standards that relate to significant aspects of the quality of those programs.* Program quality may depend on the presence or absence of specified features of programs, so some standards require the presence or absence of these features. It is more common, however, for the quality of educational programs to depend on *how well* the program's features have been designed and implemented in practice. For this reason, most of the Commission's program standards define levels of quality in program features.
- (2) *There are many ways in which a teacher preparation program could be excellent.* Different programs are planned and implemented differently, and are acceptable if they are planned and implemented well. The Commission's standards are intended to differentiate between good and poor programs. The standards do not require all programs to be alike, except in their quality, which assumes different forms in different environments.
- (3) *The curriculum of teacher education plays a central role in a program's quality.* The Commission adopts curriculum standards that attend to the most significant aspects of knowledge and competence. The standards do not prescribe particular configurations of courses, or particular ways of organizing content in courses, unless professionals on an advisory panel have determined that such configurations are essential for a good curriculum. Similarly, curriculum standards do not assign unit values to particular domains of study unless there is a professional consensus that it is essential for the Commission's standards to do so. Curriculum standards for mathematics teacher preparation are Standards 1 through 15 below.
- (4) *Teacher education programs should prepare candidates to teach the public school curriculum effectively.* The major themes and emphases of subject matter programs for teachers must be congruent with the major strands and goals of the school curriculum. It is also important for future teachers to be in a position to improve the school curriculum on the basis of new developments in the scholarly disciplines, and in response to changes in student populations and community needs. However, it is indispensable that the Commission's standards give emphasis to the subjects and topics that are most commonly taught in public schools.
- (5) *In California's public schools, the student population is so diverse that the preparation of educators to teach culturally diverse students cannot be the exclusive responsibility of professional preparation programs in schools of education.* This preparation must begin early in the collegiate experience of prospective teachers. The Commission expects subject matter programs to contribute to this preparation, and asked the Mathematics Advisory Panel to recommend appropriate program standards. The panel concurred with this request and recommended Standard 15 in this handbook.

- (6) *The curriculum of a teacher education program should be based on an explicit statement of purpose and philosophy. An excellent program also includes student services and policies such as advisement services and admission policies.* These components of teacher preparation contribute significantly to its quality; they make the program more than a collection of courses. The Commission asked the Mathematics Advisory Panel to develop standards related to (a) the philosophy and purpose of mathematics teacher preparation and (b) significant, non-curricular components of teacher preparation, to complement the curriculum standards. Again, the panel concurred, and recommended Standards 1, 16 and 18 through 21.
- (7) *The Commission is concerned about the high level of attrition among beginning teachers, and has successfully sponsored legislation to improve the conditions in which new teachers work.* Reality-based career exploration is also needed, to ensure that credential candidates are aware of the challenges of teaching before they invest heavily in professional preparation. The Commission considers subject matter preparation programs to be occasions when students should explore the realities of teaching children and adolescents in schools. The advisory panel also agreed with this principle and recommended Standard 17 on page 32.
- (8) *The assessment of each student's attainments in a teacher education program is a significant responsibility of the institution that offers the program.* This assessment should go beyond a review of transcripts to verify that acceptable grades have been earned in required and elective courses. The specific form, content and methodology of the assessment should be determined by the institution. In each credential category, the Commission's standards attend to the overall quality of institutional assessment of students in programs. Standard 20 on page 35 is consistent with these policies of the Commission.
- (9) *The Commission's standards of program quality allow quality to assume different forms in different environments.* The Commission did not ask the advisory panel to define all of the acceptable ways in which programs could satisfy a quality standard. The standards should define *how well* programs must be designed and implemented; they must not define specifically and precisely *how* programs should be designed or implemented.
- (10) *The Commission's standards of program quality are roughly equivalent in breadth and importance.* The standards are grouped in categories that are also roughly equivalent in scope. Each standard is accompanied by a rationale that states briefly why the standard is important to the quality of teacher education. The standards are written in clear, plain terms that are widely understood. The Handbook contains only three technical terms, which are defined on page 13.
- (11) *The Commission assists in the interpretation of the standards by identifying the important factors that should be considered when a program's quality is judged.* The Commission's adopted standards of program quality are mandatory; each program must satisfy each standard. "Factors to Consider" are not mandatory in the same sense, however. Instead, these factors suggest the types of questions that program reviewers ask, and the types of evidence they assemble and consider, when they judge whether a standard is met. Factors to Consider are not "mini-standards" that programs must "meet." The Commission expects reviewers to weigh the strengths and weaknesses of a program as they determine whether a program meets a standard. The Commission does not expect every program to be excellent in relation to every factor that could be considered.

- (12) *Whether a particular program fulfills the Commission's standards is a judgment that is made by professionals who have been trained in interpreting the standards.* Neither the Commission nor its professional staff make these judgments without relying on experts who are thoroughly trained in program review and evaluation. The review process is designed to ensure that subject matter programs fulfill the Commission's standards initially and over the course of time.

The Commission fulfills one of its responsibilities to the public and the profession by adopting and implementing standards of program quality and effectiveness. While assuring the public that educator preparation is excellent, the Commission respects the considered judgments of educational institutions and professional educators, and holds educators accountable for excellence. The premises and principles outlined above reflect the Commission's approach to fulfilling its responsibilities under the law.

Standards and the Availability of Qualified Mathematics Teachers

In addition to ensuring the qualifications of teachers, the Commission is concerned that there be a sufficient number of teachers. For this reason, the Commission in 1989 gave the advisory panel extensive information about mathematics teacher supply and demand in California. The panel reviewed quantitative data and anecdotal reports about:

- The numbers of new teachers of mathematics employed by California school districts, and fluctuations over time in the demand for mathematics teachers.
- The numbers of teachers receiving mathematics credentials from the Commission, and fluctuations over time in the credentialing of mathematics teachers.
- The numbers of teachers receiving *emergency* credentials to teach mathematics, and fluctuations over time in the demand for these emergency teachers of math.
- The numbers of college and university students preparing to become teachers of mathematics, and fluctuations over time in the potential supply of math teachers.
- The numbers of mathematics teachers who move into California each year after earning degrees and credentials outside of California.

The advisory panel reviewed these data carefully, and concluded that the overall supply of mathematics teachers in 1989-90 was sufficient to meet the needs of California school districts. This situation could change, of course, if student enrollments or teacher retirements increase more rapidly than expected. For this reason, the Commission will continue to monitor trends in mathematics teacher supply and demand. Moreover, there may not be a sufficient number of mathematics teachers who would accept positions in particular schools or districts, but the Commission will always have little influence over this circumstance. Given the statistical evidence that was available, the Commission asked the advisory panel to concentrate on defining the levels of quality that the Commission should require in subject matter preparation programs for future teachers of mathematics.

Analysis and Adoption of the Mathematics Program Standards

The Mathematics Teacher Preparation Advisory Panel drafted the program quality standards over the course of ten months. The standards were then reviewed and discussed by the Commission in a public meeting. The Commission distributed the draft standards to mathematics educators throughout California, with a request for comments and suggestions. The draft standards were forwarded to:

- Academic administrators of California colleges and universities;
- Chairpersons of mathematics departments in California colleges and universities;
- Deans of Education in California colleges and universities;
- Presidents of professional associations of teachers and mathematics teachers;
- Superintendents of county offices of education in California;
- Superintendents of school districts in California; and
- Math teachers, professors and curriculum specialists who asked for the document.

The Commission asked county and district superintendents to forward the document to mathematics teachers and curriculum specialists for their analysis and comments. The Commission also conducted two regional meetings (one in northern California and one in southern California) to enable mathematics educators to discuss the draft standards with members of the advisory panel.

After the period for public comments, the Commission's professional staff collated the responses to each standard, which were reviewed thoroughly by the advisory panel. The panel exercised its discretion in responding to the suggestions, and made several significant changes in the draft standards. On November 7, 1991, the advisory panel presented the completed standards to the Commission, which adopted all of the policies in this document on November 8, 1991.

New Mathematics Performance Assessments Adopted by the Commission

Since 1970, many applicants have qualified for the Single Subject Credential in Mathematics by passing a standardized test that was adopted by the Commission: the National Teachers Examination (NTE) in Mathematics. These prospective teachers of mathematics qualified for credentials without completing approved programs of subject matter study. In 1987 the Commission completed an extensive study of the validity of fifteen NTE Exams. Based on the results of this study, the Commission in 1989 asked the Mathematics Teacher Preparation and Assessment Advisory Panel to develop new specifications for assessing the subject matter competence of future teachers of mathematics.

The Commission asked the panel to design subject matter assessments that would be as parallel and equivalent as possible with the subject matter program standards. The advisory panel developed new specifications for a comprehensive test of knowledge of mathematics. The panel also developed specifications and model questions for a new examination that assesses the ability to interpret and solve complex mathematical problems. The Commission distributed the panel's proposed specifications to mathematics teachers, professors and curriculum specialists throughout California. Following an extensive review of the draft specifications, the panel made several revisions, and the completed specifications were adopted by the Commission.

Mathematics Teaching Credentials and Standards: Foreword by the Commission

The Commission awarded a contract to Educational Testing Service (ETS) to develop a new Content Area Performance Assessment in Mathematics that would match the advisory panel's specifications. On four occasions this new essay examination was pilot-tested and field-tested throughout California. Following each test, the panel examined the participants' responses and revised the test questions. The panel also developed detailed criteria for scoring candidates' responses, which were also field-tested in practice. On April 5, 1991, the Commission adopted a plan for implementing the Content Area Performance Assessment (CAPA) in California, and on July 19, 1991, the Commission adopted a passing standard for the CAPA in Mathematics. After the first administration of the new assessment, the Commission examined the impact of its passing standard on examinees.

Meanwhile, the Commission's specifications for the NTE Mathematics Test were presented to a national test development committee that was appointed by Educational Testing Service. Based on the advice of this committee, ETS developed a multiple-choice test that is part of the new *Praxis* series of professional examinations for teachers. The new Mathematics Test conforms to the Commission's specifications and will be administered throughout the nation beginning in 1993-94. *As a result of these initiatives by the Commission, all future candidates for the mathematics teaching credential will qualify by completing subject matter programs that meet standards of program quality and effectiveness, or by passing an examination and a performance assessment that are congruent with the program quality standards.*

The Commission's new specifications for the assessment of subject matter knowledge and competence are included in this handbook (pp. 37-39) to serve as a resource in the design and evaluation of subject matter programs for future mathematics teachers.

Standards for Professional Teacher Preparation Programs

The effectiveness of the mathematics curriculum in California schools does not depend entirely on the content knowledge of mathematics teachers. Another critical factor is the teachers' ability to *teach* mathematics. To address the *pedagogical* knowledge and effectiveness of mathematics teachers, the Commission in 1986 adopted new *Standards of Program Quality and Effectiveness for Professional Teacher Preparation Programs*. These thirty-two standards define levels of quality and effectiveness that the Commission expects of teacher education programs that are offered by Schools of Education. The standards originated in the published research literature on teacher education and teacher effectiveness. Approximately 1,500 educators from all levels of public and private education participated in the development of the standards during a two-year process of dialogue and advice. Since 1986 the Commission has updated the 32 standards on two occasions. The revised standards are now the basis for determining the status of professional preparation programs for Single Subject Teaching Credentials in California colleges and universities. The standards in this handbook have been designed for subject matter programs, to complement the 32 standards for programs of pedagogical preparation.

Subject Matter Standards for Prospective Elementary Teachers

In the mathematics curriculum, elementary teachers are expected to establish a foundation of knowledge, skills, and attitudes that young students need in order to learn the more advanced content that mathematics teachers offer in secondary schools. To address the preparation of future classroom teachers in elementary schools, the Commission in 1987 appointed an advisory panel to develop new *Standards of Program Quality for the Subject Matter Preparation of Elementary Teachers*. Following a thorough process of research, development, dialogue and consultation, the Commission in 1988 adopted these standards, which relate to (1) the broad range of subjects (including mathematics) that elementary teachers must learn, and (2) the essential qualities and features of programs offered in liberal arts departments. In 1989 the Commission appointed and trained two professional review teams, which have examined 73 subject matter programs for prospective elementary teachers, and have recommended 63 of these programs for approval. As a consequence of this reform initiative by the Commission, approximately twenty thousand prospective elementary school teachers are now engaged in undergraduate programs that meet professional standards of quality for the subject matter preparation of teachers.

Overview of the Mathematics Standards Handbook

This introduction to the handbook concludes with a statement by the Mathematics Advisory Panel regarding mathematics teaching and teacher preparation in California. Then Part 2 of the handbook includes the 21 standards (pp. 13-36) as well as the advisory panel's Specifications for the Subject Matter Knowledge and Competence of Prospective Teachers of Mathematics (37-39). Finally, Part 3 provides information about implementation of the new standards in California colleges and universities.

Contributions of the Mathematics Advisory Panel

The Commission on Teacher Credentialing is indebted to the Mathematics Teacher Preparation and Assessment Advisory Panel for the successful creation of *Standards of Program Quality and Effectiveness for the Subject Matter Preparation of Prospective Teachers of Mathematics*. The Commission believes strongly that the standards in this handbook will serve to improve mathematics teaching in California's public schools.

Request for Assistance from Handbook Users

The Commission periodically reviews its policies, in part on the basis of responses from colleges, universities, school districts, county offices, professional organizations and individual professionals. The Commission welcomes all comments about the policies in this handbook, which should be addressed to:

Commission on Teacher Credentialing
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Sacramento, California 95814-7000

Mathematics Teaching and Teacher Preparation: Introduction by the Mathematics Advisory Panel

As society's needs change, so must the goals of its schools. In the United States, the shift from an industrial to an information society makes it imperative for our educational institutions to reform radically the teaching and learning of mathematics.

The increasing need for a technologically competent and flexible workforce, as well as an informed electorate, calls for students to gain mathematical power. This includes students' ability to solve problems, communicate, reason, and establish connections mathematically. In this process, teachers must be sufficiently well prepared to introduce new content and use ways of teaching that are aligned with the emerging curriculum and evaluation standards for school mathematics.

Teachers of mathematics in the 21st century should be able to challenge their students to learn how to solve the problems of a rapidly advancing technological age. The most current educational research points to the importance of teachers being prepared to help students develop an enthusiastic spirit of inquiry and apply mathematics effectively. Students must be challenged with real life problems which emphasize exploring, investigating, reasoning, and communicating so they will be ready to make mathematically informed decisions in relation to their jobs, their government, and their lives.

All mathematics teachers should be comfortable with a variety of approaches and solutions to problems, and should be competent in exploring with students problems that they themselves do not know how to solve or have never seen before. Teachers should be able to:

. . . respond constructively to unexpected conjectures that emerge as students follow their own paths in approaching mathematical problems.

Everybody Counts, p. 65

Teachers should be able to make mathematical conjectures about problem situations and abstract properties and relationships, explain their reasoning, follow arguments, verify their assertions, and communicate their conclusions in a meaningful form. To attain this end, mathematics teachers *must* have a knowledge of mathematics that substantially exceeds the level of mathematics that is taught in their classrooms.

Mathematics teachers must create an environment that is conducive to learning, and that helps all students achieve their potential. In particular, teachers need to communicate the role that mathematics plays in making opportunities available to all people. Mathematics teachers should have a thorough understanding of the diverse nature of the population in California and its effects on our society. Teachers should understand the cultural and linguistic strengths of students, and should draw on this knowledge in the learning experience.

Mathematics teachers should be communicators. They must be able to express and explain mathematical ideas both orally and in written form. Equally important is the ability to understand the student's mathematical thinking, for:

No teaching can be effective if it does not respond to students' prior ideas. Teachers need to listen as much as they need to speak. They need to resist the temptation to control classroom ideas so that students can gain a sense of ownership over what they are learning. Doing this requires genuine give-and-take in the mathematics classroom, both among students and between students and teachers. The best way to develop effective logical thinking is to encourage open discussion and honest criticism of ideas.

Everybody Counts, pp. 59-60

A teacher must be able to approach mathematics as a study based upon prior knowledge and the construction of new meaning through interaction with other ideas or concrete materials. This constructivist approach to learning contrasts sharply with situations in which students passively absorb information or routines and automatically replicate those ideas in a mechanistic manner. It is based, instead, on the assumption that students construct their own mathematical understanding through: acting on concrete materials and connecting those experiences to mathematical ideas, verbally talking about the learning that is occurring and, through exploring mathematical ideas in the context of their use.

In reality, no one can teach mathematics. Effective teachers are those who can stimulate students to learn mathematics.

Everybody Counts, p. 58

Dialogue must exist among students, between students and teachers, as well as among teachers. Mathematics teachers must be knowledgeable of both curriculum and pedagogy that relate to the teaching of mathematics. The ability to discuss pedagogical and mathematical ideas with colleagues should be basic to the profession.

Teachers should be aware of a variety of investigative and questioning techniques including collaborative learning, and should be able to help students learn mathematical principles through the use of manipulatives, calculators, computers, and other available technology. This should be accomplished in a classroom environment in which students from a broad spectrum of readiness levels can succeed in mathematics.

Technology has revolutionized the way mathematics is practiced, and should influence the way it is learned. Mathematics teachers should keep abreast of continual advancements in technology, and be able to integrate technology into their mathematics program in a way that aids students in their exploration and learning of mathematics.

There is a need for ongoing dialogue across all disciplines, and teachers of mathematics should be taught in the context of other disciplines. Mathematics teachers must participate in professional activities in order to stay on the cutting edge of current thought in mathematics and mathematics education. In particular, teachers should be guided by the State of California Mathematics Framework and standards established by national professional organizations: National Council of Teachers of Mathematics, Mathematics Association of America, National Research Council Board on Mathematical Sciences, etc.

One of the chief assessors of student progress is the classroom teacher. Assessment, as defined by the National Research Council, is the:

. . . mechanism whereby teachers can learn how students think about mathematics as well as what students are able to accomplish Because assessment is so pervasive and has such powerful impact on the lives of both students and teachers, it is very important that assessment practice align properly both with the purpose of the test and with curricular objectives.

Everybody Counts, pp. 69-70

Teachers should be able to assess students, programs, and themselves using a variety of evaluative tools which are aligned with state framework curricular objectives.

Since teachers teach much as they were taught, university courses for prospective teachers must exemplify the highest standards for instruction. . . Very few teachers have had the experience of constructing for themselves any of the mathematics that they are asked to teach, of listening to students who are developing their own mathematical understandings, or of guiding students to their own discovery of mathematical insights. Prospective teachers should learn mathematics in a manner that encourages active engagement with mathematical ideas.

Everybody Counts, pp. 65-66

Prospective teachers must be given opportunities to learn mathematics in a style that is consistent with the ways in which they will be expected to teach.

The need to improve the state of mathematics education is apparent. All involved must work to improve both the content and the process of teaching mathematics, curricular standards, assessment practices, and the teaching profession. The teacher preparation standards in this document will assist in meeting the challenges of the 21st century in California. The standards will lead to reforms in the preparation of mathematics teachers by providing broad as well as specific directions for change.

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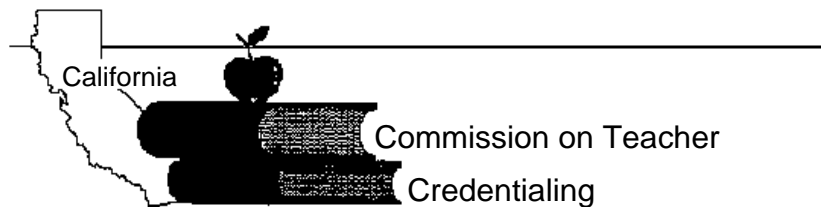
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Part 2

Standards of Program Quality and Effectiveness in Mathematics



Definitions of Key Terms

Standard

A "standard" is a statement of program quality that must be fulfilled for initial or continued approval of a subject matter program by the Commission. In each standard, the Commission has described an acceptable level of quality in a significant aspect of mathematics teacher preparation. The Commission determines whether a program satisfies a standard on the basis of an intensive review of all available information related to the standard by a review panel whose members (1) have expertise in mathematics teacher preparation, (2) have been trained in the consistent application of the standards, and (3) submit a recommendation to the Commission regarding program approval.

The Commission's adopted Standards of Program Quality and Effectiveness for Subject Matter Programs in Mathematics begin on page 15. The Commission's authority to establish and implement the standards derives from Section 44259 (b) (5) of the California Education Code.

Factors to Consider

"Factors to consider" guide program review panels in judging the quality of a program in relation to a standard. Within the scope of a standard, each factor defines a dimension along which programs vary in quality. The factors identify the dimensions of program quality that the Commission considers to be important. To enable a program review panel to understand a program fully, a college or university may identify additional quality factors, and may show how the program fulfills these added indicators of quality. In determining whether a program fulfills a given standard, the Commission expects the review panel to consider all of the related quality factors in conjunction with each other. In considering the several quality factors for a standard, excellence on one factor compensates for less attention to another indicator by the institution. For subject matter programs in mathematics, the adopted factors to consider begin on page 16.

Precondition

A "precondition" is a requirement for initial and continued program approval that is based on California state laws or administrative regulations. Unlike standards, preconditions specify requirements for program compliance, not program quality. The Commission determines whether a program complies with the adopted preconditions on the basis of a program document provided by the college or university. In the program review sequence, a program that meets all preconditions is eligible for a more intensive review to determine if the program's quality satisfies the Commission's standards. Preconditions for the approval of subject matter programs in mathematics are on page 14 of this handbook. Details regarding the program review sequence are on pages 45-54.

Preconditions for the Approval of Subject Matter Programs in Mathematics

The following Preconditions for the Approval of Subject Matter Programs in Mathematics are based on California Administrative Code Sections 80085.1 and 80086. The Commission's statutory authority to establish and enforce the preconditions is based on Sections 44310 through 44312 of the California Education Code.

- (1) Each Program of Subject Matter Preparation for the Single Subject Teaching Credential in Mathematics shall include (a) at least 30 semester units (or 45 quarter units) of core mathematics coursework that is directly related to subjects that are commonly taught in departmentalized mathematics classes in California public schools, and (b) a minimum of 15 semester units (or 22 quarter units) of coursework that provides breadth and perspective to supplement the essential core of the program. These two requirements are elaborated in Preconditions 2 and 3.
- (2) The basic core of the program shall include coursework in (or directly related to) first and second year algebra (or demonstrated proficiency), geometry, first and second year calculus, number theory, mathematical systems, statistics and probability, discrete mathematics, and the history of mathematics.

In addition to describing how a program meets each standard of program quality in this handbook, the program document by an institution shall include a listing and catalog description of all courses that constitute the basic core of the program. Institutions shall have flexibility to define the core in terms of (a) specifically required coursework or (b) elective courses related to each required mathematical subject. Institutions may also determine whether the core consists of (a) one or more distinct courses for each mathematical subject, or (b) courses that offer integrated coverage of the required subjects.

- (3) Additional coursework in the program shall be designed to provide breadth and perspective to supplement the essential core of the program.

A program document shall include a listing and catalog description of all courses that are offered for the purposes of breadth and perspective. Institutions may define this program component in terms of required coursework or elective courses.

Coursework offered by any appropriate department(s) of a regionally accredited institution may satisfy the preconditions and standards in this handbook.

Preamble to the Mathematics Program Standards

The Commission and the Mathematics Advisory Panel believe that the goals of mathematics education are mathematical power and literacy, which include the ability to solve problems, communicate ideas, reason, make mathematical connections, and use current technology, as well as understanding the concepts and topics that have traditionally composed the mathematics curriculum. The development of mathematical power and literacy requires a redefinition of the mathematics curriculum so mathematics is approached from a unified perspective and not as a series of disjointed topics, specific concepts, or procedures to be followed.

A successful Mathematics Subject Matter Preparation Program must be aligned with this redefinition of the curriculum. The program must approach mathematics in a unified, integrated way, and must provide opportunities for students to learn mathematics in ways that are consistent with the curriculum they will be expected to teach in their own classrooms.

The Mathematics Advisory Panel developed the 21 Standards of Program Quality and Effectiveness in the following pages. The panel conceptualized four sets of standards, and defined the following relationships among the four sets of standards.

- (1) Standards 1 through 6 relate to the overall content of a subject matter program in mathematics. These standards describe qualities that must characterize the curriculum as a whole, regardless of how the content of mathematics is organized in the program.
- (2) Standards 7 through 14 focus on specific mathematical content that students must learn to become competent in mathematics. Each standard is not intended to be the focus of a separate course, however. Regardless of how the subjects of mathematics are conceptualized and organized, the program must emphasize the underlying linkages and relationships among these subjects, to underscore the fact that mathematics is a unified body of knowledge.
- (3) Standards 15 through 17 relate to essential features or characteristics of a subject matter program's *curriculum*. These standards relate to the context in which students can understand the mathematics they are learning as well as ideas about how people learn mathematics.
- (4) Standards 18 through 21 describe essential *non-curricular* features of a subject matter program for prospective teachers of mathematics. The standards in this set define critical responsibilities of an institution that offers subject matter preparation for students who plan to teach mathematics.

Overall, the scope, content, themes and emphases of the subject matter program should be generally congruent with the specifications for subject matter knowledge and competence for prospective teachers of mathematics on pages 37 through 39 of this handbook.

Standards of Program Quality and Effectiveness

Category 1: Curriculum and Content of the Program

Standard 1: Program Philosophy and Purpose

The subject matter preparation program in mathematics is based on an explicit statement of program philosophy that expresses its purpose, design and desired outcomes, and defines the institution's concept of a well-prepared teacher of mathematics. The program philosophy, design and desired outcomes are appropriate for preparing students to teach mathematics in California schools.

Rationale for Standard 1

To ensure that a subject matter program is appropriate for future teachers, it should have an explicit statement of philosophy which expresses the institution's concept of a well-prepared teacher of mathematics. This statement provides direction for program design and it assists the faculty in identifying program needs and emphases, developing course sequences and conducting program reviews. The philosophy statement also informs students of the basis for program design, and communicates the institution's aims to school districts, prospective faculty members and the public. The responsiveness of a program's philosophy, design and desired outcomes to the contemporary conditions of California schools are critical aspects of its quality.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- The program philosophy, design and desired outcomes are collectively developed by participating faculty, reflect an awareness of recent research and thinking in the discipline of mathematics, and are consistent with each other.
- The program philosophy is consistent with the major themes and emphases of the California State Curriculum Framework, other state curriculum documents, and nationally adopted guidelines for teaching mathematics.
- The statement of program philosophy shows a clear awareness of the preparation that students need in order to teach mathematics effectively among diverse students in California schools.
- Expected program outcomes for students are defined clearly so student assessments and program reviews can be aligned appropriately with program goals.
- The institution periodically reviews and reconsiders the program philosophy, design and intended outcomes in light of ongoing research and thinking in the discipline, nationally accepted standards and recommendations, and the changing needs of public schools in California.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 2

Mathematics as Problem Solving

Each program requires students to use a variety of strategies to formulate and solve appropriate problems.

Rationale for Standard 2

In daily life, individuals are confronted with problems to solve. Prospective teachers of mathematics need to develop effective strategies for solving real world problems and responding to non-routine situations. These challenges should occur throughout a program of subject matter preparation for prospective teachers of mathematics.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- The program requires students to examine given situations, extract quantitative information abstractly from these descriptions, and formulate appropriate problems related to the given situations.
- The program involves students in using a variety of mathematical models to represent problem situations, including the identification of simplifications or assumptions made in creating mathematical models and their effect on the solutions.
- The program requires students to interpret the results of a solution in the context of the given situation.
- The program fosters a spirit of inquiry, excitement, and perseverance in students.
- The program requires students to generalize solutions where appropriate and justify their conclusions.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 3

Mathematics as Communication

Each program requires students to use language and mathematical symbols to communicate mathematical ideas.

Rationale for Standard 3

The ability to communicate precisely is an extremely important skill in all disciplines. Mathematics is an essential tool in effective communication. Prospective teachers must be able to communicate mathematical ideas in multiple ways, to facilitate learning of this subject by future students in the schools. Communication skills should therefore be developed in conjunction with mathematical literacy in each major component of a subject matter program.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- The program requires students to be able to write and speak about mathematical ideas, using appropriate mathematical terminology.
- The program provides opportunities for students to communicate mathematical ideas at a variety of levels.
- The program requires students to communicate mathematical information in various forms, including charts, graphs, tables, and figures.
- The program provides opportunities for students to use clarifying and extending questions to learn mathematics and to communicate mathematical ideas.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 4

Mathematics as Reasoning

Each program requires students to demonstrate a variety of reasoning skills.

Rationale for Standard 4

Reasoning is fundamental to knowing and doing mathematics. It is essential that an emphasis on reasoning pervade all mathematical activity, to give individuals access to mathematics as a powerful way of making sense of the world.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- The program develops students' ability to reason inductively and deductively.
- The program requires students to understand and apply a variety of reasoning processes such as spatial, probabilistic, and proportional processes.
- The program develops students' ability to formulate and test conjectures, construct counter-examples, make valid arguments, and judge the validity of mathematical arguments.
- The program requires students to evaluate the reasonableness of solutions to problems.
- The program develops students' number sense and estimation skills.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 5

Mathematical Connections

Each program includes investigation of the connections and interplay among various mathematical topics and their applications that cover a broad range of phenomena across appropriate disciplines.

Rationale for Standard 5

Knowledge of internal and external mathematical connections are of fundamental importance for the development of mathematical understanding and the use of mathematics in the real world. Relationships among mathematical subjects and applications must be a consistent theme of a subject matter program's curriculum.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- The program requires students to be able to use applications when explaining mathematical concepts.
- The program requires students to be able to show how mathematical topics are inter-related.
- The program requires students to apply mathematical thinking and modeling to solve problems that arise in other disciplines.
- The program develops students' ability to recognize how a given mathematical model can represent a variety of situations.
- The program develops students' ability to create a variety of models to represent a single situation.
- The program develops students' awareness of the presence of mathematics in the world around us.
- The program requires students to show evidence of their knowledge of the interconnectedness of topics of mathematics from a historical perspective.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 6

Mathematics with the Use of Technology

Each program incorporates current technology that is appropriate for the learning of mathematics.

Rationale for Standard 6

Technological developments are affecting our daily lives at an ever increasing rate. Technology provides opportunities for the investigation, development, understanding, and communication of mathematical concepts. It has changed the very nature of the problems that are important to mathematics, as well as the methods mathematicians use to investigate them. Learning to use technology as a tool for learning mathematics should be a pervasive characteristic of a subject matter program for teachers.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- The program provides opportunities for students to use technological tools, such as computers, calculators, graphing utilities, video, and interactive programs, to learn concepts, explore new theories, conduct investigations, make conjectures, and solve problems.
- The program provides opportunities for students to analyze, compare, and evaluate the appropriateness of technological tools and their uses in mathematics.
- The program utilizes appropriate technological tools when providing instruction and assessing students in mathematics.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 7

Algebra

Each program requires students to have an understanding of the foundations of high school algebra from an advanced standpoint, and linear and matrix algebra.

Rationale for Standard 7

Algebra is a fundamental language through which mathematics is communicated. It is also a commonly-taught subject in mathematics classes in California. Fulfillment of Standard 7 is essential for each prospective teacher of mathematics.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which

- The program requires students to understand the different uses of variables and the power of mathematical abstraction and symbolism.
- The program requires students to understand a variety of algebraic techniques used to analyze mathematical situations.
- The program requires students to use a variety of algebraic representations to model problem situations.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 8

Geometry

Each program requires students to have a fundamental knowledge of geometry and its relation to algebra.

Rationale for Standard 8

Geometry is a vehicle for studying axiomatic systems, for representing real world phenomena, and for visualizing mathematical ideas. It is also a commonly-taught subject in California mathematics classes. A fundamental knowledge of geometry, and its relation to algebra, is essential for each teacher of mathematics.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- The program requires students to translate between synthetic and coordinate representations.
- The program requires students to develop an understanding of axiomatic systems through the use of geometry.
- The program emphasizes that geometry is derived from and applies to the real world in both aesthetic and practical ways.
- The program provides students with a foundation in non-Euclidean geometries.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 9

Functions and Calculus

Each program includes the study and use of functions and calculus.

Rationale for Standard 9

The concepts of functions and calculus are important unifying ideas in mathematics. Functions and calculus are also commonly taught in mathematics classes, so it is essential that subject matter programs include these studies.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- The program requires students to model real world problems with a variety of algebraic and transcendental functions.
- The program requires students to translate between the tabular, symbolic, and graphical representations of functions.
- The program requires students to understand the basic concepts of calculus, including the derivative, integral, differential equations, their interconnections, and their use in analyzing and solving real-world problems.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 10

Number Theory

Each program requires students to have an understanding of number theory.

Rationale for Standard 10

Number theory, which illustrates the beauty of pure mathematics, provides a familiar context for mathematical explorations.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- The program involves students in formulating conjectures about natural numbers.
- The program requires students to verify hypotheses through inductive and deductive proofs.
- The program requires students to understand advanced topics such as diophantine equations, number-theoretic functions, quadratic reciprocity, primitive roots, and continued fractions.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 11

Mathematical Systems

Each program provides students with an understanding of the nature and purpose of axiomatic systems, and the ability to prove fundamental theorems utilizing various mathematical systems.

Rationale for Standard 11

In order to understand the structure and logic inherent in mathematics, a foundation in mathematical systems is essential.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which

- The program requires students to understand symbolic logic.
- The program requires students to have a foundation in the structure of the real number system.
- The program requires students to understand the theory of and operations within groups, rings, and fields.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 12

Statistics and Probability

Each program provides students with an understanding of statistics and probability.

Rationale for Standard 12

Statistics and probability are encountered daily. There is an increasing need for all citizens to organize and interpret data. Probability and statistical inference are fundamental to the study of many different disciplines, and are essential elements in the subject matter background of mathematics teachers.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- The program requires students to design statistical experiments in which they collect, interpret, chart, graph, and justify their findings.
- The program requires students to draw inferences from charts, tables, and graphs that represent real world situations.
- The program requires students to have an understanding of probability, including dependent and independent events, and discrete and continuous probability distributions such as normal, binomial, and chi-square.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 13

Discrete Mathematics

Each program provides students with an understanding of discrete mathematics.

Rationale for Standard 13

Applications of discrete mathematics are expanding rapidly as the computer becomes a fundamental tool for analyzing important societal problems.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- The program requires students to represent and solve problems using such techniques as graph theory, matrices, sequences, linear programming, difference equations, combinatorics, computer science, and other topics and tools.
- The program requires students to develop algorithms, including computer programming, rather than merely applying algorithms or programs to given problems.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 14

History of Mathematics

Each program requires students to have a foundation of knowledge about the history of mathematics, and a historical perspective regarding the development of mathematics.

Rationale for Standard 14

A foundation in the history of mathematics enables students to gain a rich understanding of the origins of mathematical concepts.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which

- The program requires students to understand the chronological and topical development of mathematics.
- The program requires students to understand the contributions of historical figures, including individuals of various racial, ethnic, gender, and national groups.
- The program requires students to understand the contributions of mathematics to society, and its impact on society.
- The program provides opportunities for students to be exposed to the mathematical discoveries that have affected the course of civilization.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 15

Equity and Diversity in the Program

The subject matter preparation program promotes educational equity by utilizing instructional, advisement and curricular practices that offer equal access to program content and career options for all students. Each student in the program acquires knowledge, understanding and appreciation of the perspectives and contributions of diverse cultural, ethnic and gender groups related to the discipline.

Rationale for Standard 15

Students who attend California schools are increasingly diverse. They live in a society that has benefitted from the perspectives and contributions of men, women, and many cultural, ethnic and gender groups. Prospective teachers must understand and appreciate the cultural perspectives and intellectual contributions of these groups. They must also be aware of barriers to academic participation and success, and must encounter equitable practices of education during their preparation.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- In the course of the program, students experience classroom practices and use instructional materials that promote educational equity among diverse learners.
- The program includes faculty role models from diverse cultural and ethnic groups, men and women, and individuals with exceptional needs.
- The program includes faculty who are concerned about and sensitive to diverse cultural and ethnic groups, men, women, and individuals with exceptional needs.
- The institution encourages men and women students, and students who are culturally and ethnically diverse, to enter and complete the subject matter program.
- The program provides knowledge and enhances understanding and appreciation of the cultural dimensions and context of mathematics as a subject of study.
- Each student learns about the contributions and perspectives of diverse cultural, ethnic and gender groups related to significant mathematical subjects.
- Students examine ways in which the growth and development of the discipline have affected different cultural, ethnic, gender and handicapped groups.
- Coursework in the program fosters understanding, respect and appreciation of human differences, including cultural, ethnic, gender and language variations in the learning of mathematics.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 16

Delivery of Instruction in the Program

Each program utilizes multiple instructional strategies, activities and materials that are appropriate and effective for mathematics instruction. Candidates examine ways in which mathematical knowledge is transformed for use in practical applications.

Rationale for Standard 16

The content and delivery of instruction in mathematics are inseparable. Students must be given opportunities to learn mathematics in styles that are consistent with the ways in which they will be expected to teach. In order for prospective teachers to be able to connect mathematics with the lives of their students, they must learn early about ways in which mathematical knowledge can be reconceptualized for practical applications.

Factors to Consider

When reviewers judge whether a program meets this standard the Commission expects them to consider the extent to which:

- The program emphasizes learning to *understand* mathematics, not following rules and procedures.
- The program provides opportunities for students to experience a variety of instructional formats such as small collaborative groups, individual explorations, peer instruction, and whole class discussions facilitated by the students.
- The program provides opportunities for students to be actively involved in learning mathematics through tactile, visual, and auditory modalities.
- The program provides opportunities for students to develop and reinforce mathematical concepts and skills through open-ended situational lessons.
- The program provides opportunities for students to understand ways in which assessment can be connected with instruction through use of portfolios, group and individual performance tasks, observations and interviews.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 17

Field Experiences in the Program

Each program requires that students participate in field experiences related to mathematics.

Rationale for Standard 17

To fully understand mathematics as a prospective teacher of the subject, students must observe others learning, doing, or using mathematics.

Factors to Consider

When reviewers judge whether a program meets this standard the Commission expects them to consider the extent to which:

- The program provides opportunities for students to have experiences in schools, including institutions of higher education.
- The program provides opportunities for students to have other off-campus field experiences related to mathematics.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Category II: Essential Features of Program Quality

Standard 18

Coordination of the Program

The subject matter preparation program is coordinated effectively by one or more persons who are responsible for program planning, implementation and review.

Rationale for Standard 18

The accomplishments of students in a subject matter program depend in part on the effective coordination of the program by responsible members of the institution's administrative staff and/or academic faculty. For students to become competent in the subjects they will teach, all aspects of their subject matter preparation must be planned thoughtfully, implemented conscientiously and reviewed periodically by designated individuals.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- There is effective communication and coordination among the academic program faculty; and between the faculty and local school personnel, local community colleges, and the professional education faculty.
- One or more persons are responsible for overseeing and assuring the effectiveness of student advisement and assessment in the program (refer to Standards 19 and 20), and of program review and development by the institution (refer to Standard 21).
- Sufficient time and resources are allocated for responsible faculty and/or staff members to coordinate all aspects of the program.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 19

Student Advisement and Support

A comprehensive and effective system of student advisement and support provides appropriate and timely program information and academic assistance to students and potential students, and gives attention to transfer students and members of groups that traditionally have been underrepresented among teachers of mathematics.

Rationale for Standard 19

To become competent in a discipline of study, students must be informed of the institution's expectations, options and requirements; must be advised of their own progress toward academic competence; and must receive information about sources of academic and personal assistance and counseling. Advisement and support of prospective teachers are critical to the effectiveness of subject matter preparation programs, particularly for transfer students and members of groups that traditionally have been underrepresented in the discipline. In an academic environment that encourages learning and personal development, prospective teachers acquire a student-centered outlook toward education that is essential for their subsequent success in public schools.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- Advisement and support in the program are provided by qualified individuals who are assigned those responsibilities, and who are available and attentive when the services are needed.
- Advisement services include information about course equivalencies, financial aid options, admission requirements in professional preparation programs, state certification requirements, field experience placements, and career opportunities.
- Information about program purposes, options and requirements is available to prospective students and distributed to enrolled students.
- The institution encourages students to consider careers in teaching, and attempts to identify and advise interested individuals in appropriate ways.
- The institution actively seeks to recruit and retain students who are members of groups that traditionally have been underrepresented among mathematics teachers.
- The institution collaborates with community colleges to articulate academic curricula and to facilitate the transfer of students into the subject matter program.
- The program has other qualities related to this standard that are brought to the attention of the team by the institution.

Standards for Mathematics Teacher Preparation

Standard 20

Assessment of Subject Matter Competence

The program uses multiple measures to assess the subject matter competence of each student formatively and summatively in relation to the content of Standards 1 through 14. The scope and content of each student's assessment is congruent with the studies the student has completed in the program.

Rationale for Standard 20

An institution that offers content preparation for prospective teachers has a responsibility to verify their competence in the subject(s) to be taught. It is essential that the assessment in mathematics use multiple measures, have formative and summative components, and be as comprehensive as Standards 1-14. Its content must be congruent with the studies that each student actually pursues in the program. Course grades and other course evaluations may be part of the assessment, but may not comprise it entirely.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- The assessment process includes a variety of approaches, such as student performances, presentations, projects, portfolios, observations and interviews, as well as oral and written examinations based on criteria established by the institution.
- The assessment encompasses the content of Standards 1-14, and is congruent with each student's actual studies in the program.
- The scope, content, themes and emphases of the assessment are generally congruent with the specifications for subject matter knowledge and competence for prospective teachers of mathematics on pages 37 through 39 of this handbook.
- The assessment process is valid, reliable, equitable, and fair, and includes provisions for student appeals.
- The assessment scope, process and criteria are clearly delineated and made available to students.
- The institution makes and retains thorough records regarding each student's performance in the assessment.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Standards for Mathematics Teacher Preparation

Standard 21

Program Review and Development

The subject matter program has a comprehensive, ongoing system of review and development that involves faculty, students and appropriate public school personnel, including mathematics teachers, and that leads to continuing improvements in the program.

Rationale for Standard 21

The continued quality and effectiveness of subject matter preparation depends on periodic reviews and improvements of the programs. Program development and improvement should be based in part on the results of systematic, ongoing reviews that are designed for this purpose. Reviews should be thorough, and should include multiple kinds of information from diverse sources.

Factors to Consider

When reviewers judge whether a program meets this standard, the Commission expects them to consider the extent to which:

- Systematic and periodic reviews of the subject matter program reexamine its philosophy, purpose, design, curriculum and intended outcomes for students.
- Information is collected about the program's strengths, weaknesses, and needed improvements from participants in the program, including faculty, students, recent graduates, and employers of recent graduates, and from other appropriate public school personnel, including teachers of mathematics.
- Program development and review involves consultation among departments that participate in the program, including subject matter and education departments, and includes review of recommendations by elementary, secondary and community college educators.
- Program improvements are based on the results of periodic reviews, the implications of new knowledge in the discipline of mathematics, the identified needs of students and school districts in the region, and recent curriculum decisions of the state in the area of mathematics.
- Assessments of students (pursuant to Standard 20) are also reviewed and used for improving the philosophy, structure, curriculum and/or outcome expectations of the program.
- The program has other qualities related to this standard that are brought to the reviewers' attention by the institution.

Specifications for the Subject Matter Knowledge and Competence of Prospective Teachers of Mathematics

Mathematics Teacher Preparation and Assessment Advisory Panel Commission on Teacher Credentialing 1991

A student who seeks to earn the Single Subject Teaching Credential in Mathematics should demonstrate mathematical power. To verify that mathematical power has been attained, the Commission has developed and adopted a standardized subject matter assessment in mathematics, which consists of two sections: a two-hour examination of multiple-choice questions, and a two-hour performance assessment in mathematics. In both sections, the problems require understanding of mathematical abstraction and symbolism as well as mathematical relationships. Examinees are allowed to use hand-held calculators of their own choosing.

The Mathematics Teacher Preparation and Assessment Advisory Panel drafted the following specifications of content domains for the two sections of the assessment. The specifications illustrate the mathematical knowledge, skills and abilities that students should acquire and develop in a subject matter program for prospective teachers of mathematics.

Descriptions of Content Domains

Percent of Test

Algebra

10

The standardized assessment includes the use of operations and expressions in the solution of algebraic equations, including linear and matrix algebra. It also includes a variety of algebraic representations to model situations. Algebraic skills are embedded in the context of problem solving. Problems are neither trivial nor focused on operations alone.

Geometry

15

Problems on the standardized assessment require use of axiomatic systems and analytical systems, and understanding of two and three dimensional geometry, including concepts of coordinate, synthetic, non-Euclidian and transformational geometry.

Functions

25

Problems on the assessment require the use of algebraic and transcendental functions, the use of power series, the use of limits, calculus, and differential equations.

Specifications: Subject Matter Knowledge and Competence of Mathematics Teachers

<u>Descriptions of Content Domains</u>	<u>Percent of Test</u>
Number Theory	10
Problems allow for formulations and the use of conjectures about natural numbers and the verification of hypotheses through inductive and deductive proofs. These problems include (but are not limited to) problems using Diophantine equations, Chinese remainder theorems, and congruence classes.	
Mathematical Systems	10
Items require the use of fundamental properties of the real and complex number systems, Boolean Algebra, and symbolic logic.	
Statistics and Probability	15
Charts, tables, and graphs permit drawing inferences. Also included are probability distributions, including normal curves, binomials, chi squares, central tendencies, and dispersion.	
Discrete Mathematics	10
These problems include the use of matrices, sequences, combinatorics, graph theory, linear programming, difference equations, and applications of computer science.	
History of Mathematics	5
Problems address mathematical discoveries, their chronological development, and their impact on human society and thought.	

Criteria for the Selection of Examination Questions

Potential questions for the assessment are judged in relation to the following criteria.

- (1) Skills to be tested are embedded in problem solving situations.
- (2) Questions permit the use of calculators.
- (3) Open-ended problems permit a variety of solution strategies.
- (4) Items are non-routine and require more than simple calculations alone.
- (5) Solutions involve techniques from more than one content area of mathematics.
- (6) Items require generalized solutions where appropriate and justified conclusions.

Matrix of Mathematical Skills and Question-Response Types

The standardized assessment of mathematical power is comprehensive in content and varied by the kinds of responses that examinees are required to make.

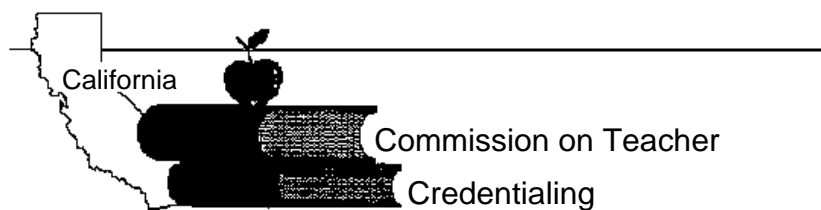
Response Types

Skills

	Recall	Categorical Understanding	Problem Solving	Applications
1. Multiple Choice a. Easy b. Moderate c. Difficult				
2. Short Answer (Constructed Response) a. Easy b. Moderate c. Difficult				
3. Extended Response a. Easy b. Moderate c. Difficult				

Part 3

Implementation of Mathematics Teaching Standards



Implementation of Program Quality Standards for the Subject Matter Preparation of Mathematics Teachers

The Program Quality Standards for Subject Matter Preparation in Mathematics are part of a broad shift in the policies of the Commission on Teacher Credentialing related to the preparation of professional teachers and other educators in California colleges and universities. The Commission initiated this policy change to foster greater excellence in educator preparation, and to combine flexibility with accountability for institutions that offer programs for prospective teachers. The success of this reform effort depends on the successful *implementation* of program quality standards for each credential.

Pages 41 through 44 of the handbook provide general information about the transition to program quality standards for all teaching credentials. Then the handbook provides specific information about implementation of the mathematics standards (pp. 45-54).

Transition to Quality Standards for All Teaching Credentials

The Commission is gradually developing and implementing Standards of Program Quality and Effectiveness for all teaching credentials. For subject matter programs, this process began in 1986, with the appointment of an expert advisory panel in elementary education, which was asked to develop *Standards of Program Quality for the Subject Matter Preparation of Elementary Teachers*. In 1988 the Commission adopted these standards for the Multiple Subjects Teaching Credential. The standards have been implemented in 55 colleges and universities, which offer a total of 64 programs.

In 1989, the Commission established five subject matter advisory panels to develop standards for the subject matter preparation of prospective secondary teachers in English, mathematics, life science, physical science and social science. The panels consisted of subject matter experts from throughout California: K-12 teachers of the subjects, public school curriculum specialists, university professors of the subjects, and other subject matter specialists.

In 1991 the Commission established four more panels to develop program standards in art, music, foreign languages and physical education. Draft standards developed by these panels are being reviewed by colleges, universities, professional organizations, and local and state education agencies, prior to being completed by the panels and adopted by the Commission. Implementation of these standards will follow a timeline similar to the milestones displayed on page 48 of this handbook.

In 1993, the Commission plans to appoint advisory panels to develop program standards in business education, health education, home economics, and industrial technology. Initial drafts of standards in these subjects will be distributed widely for discussion and comment before they are completed by the panels and adopted by the Commission. Again, implementation will follow a timeline like that on page 48.

Alignment of Program Standards and Performance Assessments

The Teacher Preparation and Licensing Act of 1970 (Ryan Act) established the requirement that candidates for teaching credentials verify their knowledge of the subjects they intend to teach. Candidates for teaching credentials may satisfy the subject matter requirement by completing approved subject matter programs or passing subject matter assessments that have been adopted by the Commission. The Commission is concerned that the scope and content of the subject matter assessments be aligned and congruent with the program quality standards in each subject.

To achieve this alignment and congruence in mathematics, the Commission asked the Mathematics Teacher Preparation and Assessment Advisory Panel to develop subject matter assessment specifications that would be consistent in scope and content with the program quality standards in this handbook. Following extensive discussion and review, the Commission adopted a detailed set of *Specifications for the Assessment of Subject Matter Knowledge and Competence of Prospective Teachers of Mathematics*, which are in pages 37-39. College and university faculty and administrators are urged to examine these specifications as a source of ideas and information about content that is important to include in subject matter programs.

The Commission seeks to align the assessment specifications with the program standards in each subject area. Each subject matter advisory panel is asked to develop standards and specifications that are as congruent with each other as possible, to maximize the equivalence between credentials that are earned by completing programs and ones that are earned by passing examinations.

Validity and Authenticity of Subject Matter Assessments

The Commission is also concerned that the subject matter assessments of prospective teachers address the full range of knowledge, skills and abilities needed by teachers of each subject. For fifteen years the Commission relied on subject matter examinations that consisted entirely of multiple-choice questions. In 1987-88, the Commission evaluated fifteen of these subject matter exams comprehensively. More than 400 teachers, curriculum specialists and university faculty examined the specifications of these tests, as well as the actual test questions. An analysis of the reviewers' aggregated judgments showed that (1) particular changes were needed in each multiple-choice test, and (2) each multiple-choice test should be supplemented by a performance assessment in the subject.

Since 1988, the Commission's subject matter advisory panels have created Content Area Performance Assessments (CAPAs) for each of ten Single Subject Credentials. The CAPAs consist of problems, questions and exercises to which examinees construct complex responses, instead of selecting an answer among four given answers. Examinees' responses are scored on the basis of specific criteria that were created by the advisory panels and are administered by subject specialists who are trained in the scoring process. Candidates for the ten Single Subject Credentials must pass a CAPA as well as a multiple-choice test of their subject matter knowledge, unless they complete an approved subject matter program. Meanwhile, for the Multiple Subject Credential, the Commission has developed and adopted a new exam (the MSAT) that consists of a Breadth of Knowledge Examination (2 hours) and a Content Skills Assessment (3 hours). By developing and adopting the CAPA and MSAT assessments, the Commission has committed itself to assessing the subject matter knowledge and competence of prospective teachers as authentically and comprehensively as possible.

New Terminology for "Waiver Programs"

In enacting the Ryan Act, the Legislature clearly regarded the successful passage of an adopted examination as the principal way to meet the subject matter requirement. However, the law also allowed candidates to complete Commission-approved subject matter programs to "waive" the examination. Because of this terminology in the 1970 statute, subject matter programs have commonly been called "waiver programs" throughout California.

In reality, the law established two alternative ways for prospective teachers to meet the subject matter requirement. An individual who completes an approved subject matter program is not required to pass the subject matter examination, and an individual who achieves a passing score on an adopted exam is not required to complete a subject matter program. Overall, the two options are used by approximately equal numbers of candidates for initial teaching credentials. Subject matter programs are completed by more than half of the candidates for Single Subject Credentials, but the adopted examination is the preferred route for more than half of all Multiple Subject Credential candidates.

Because of the significant efforts of the Commission and its expert advisory panels, subject matter programs and examinations are being made as parallel and equivalent to each other as possible. The term "waiver programs" does not accurately describe a group of programs that are *alternatives to* subject matter examinations. For this reason, the Commission uses the term "subject matter programs" instead of "waiver programs," which is now out of date.

Improvements in the Review of Subject Matter Programs

Some individuals who are involved in the subject matter preparation of prospective teachers will recall the subject matter program reviews that were done by "Waiver Program Panels" for the Commission beginning in 1983. Although there are some similarities between the "old" policies and the plan for implementing the "new" standards in this handbook, there are also some major changes.

- (1) The standards are much broader than the prior guidelines for subject matter programs. The standards provide considerably more flexibility to institutions.
- (2) As a set, the standards are more comprehensive in addressing the quality of subject matter preparation. They provide a stronger assurance of excellent preparation.
- (3) The new Program Review Panels conduct more intensive reviews that focus on program quality issues rather than course titles and unit counts.
- (4) The new panels have more extensive training because the standards require that they exercise more professional discretion regarding the quality of programs.
- (5) Institutional representatives meet with the Review Panels to discuss questions about programs and standards. Improved communications lead to better decisions.

The Commission welcomes comments and suggestions about the program review process, which should be addressed to the Executive Director.

Ongoing Review and Approval of Subject Matter Programs

After the Commission grants full or interim approval to subject matter programs, the programs will be reviewed at six-year intervals, in approximately the same way as the Commission reviews professional preparation programs in California universities and colleges. Periodic reviews will be based on the Standards of Program Quality and Effectiveness. Like professional preparation programs, subject matter programs will be reviewed onsite by small teams of trained reviewers. Reviewers will acquire information about program quality from institutional documents and interviews with program faculty, administrators, students, and recent graduates. Prior to each review, the Commission will provide detailed information about its scope, methodology, potential benefits and other implications for the college or university.

Review and Improvement of Subject Matter Standards

Beginning in 1997-98 the Commission will begin a cycle of review and reconsideration of the *Standards of Quality and Effectiveness for Subject Matter Programs in Mathematics* and in other subjects. The standards will be reviewed and reconsidered in relation to changes in academic disciplines, school curricula, and the backgrounds and needs of California students (K-12). Reviews of program standards will be based on the advice of subject matter teachers, professors and curriculum specialists. Prior to each review, the Commission will invite interested individuals and organizations to participate in it. If the Commission modifies the mathematics standards, an amended handbook will be forwarded to each mathematics department with an approved program.

Mathematics Teacher Preparation: Adoption and Implementation of Standards by the Commission

The Mathematics Teacher Preparation Advisory Panel completed its work on the Standards of Program Quality and Effectiveness in 1991. The Commission was prepared to adopt and implement the panel's work, but was concerned about the fiscal impact of the standards during a budget crisis. On November 8, 1991, the Commission adopted the standards, but continued to be concerned about their potential fiscal impact on colleges and universities. Commissioners directed the staff to monitor the crisis and present a plan for implementing the standards in ways that would be fiscally feasible for institutions.

The budgets of postsecondary institutions continued to decline during 1992. On October 1, 1992, the Commission's professional staff recommended an implementation plan for the standards that would accommodate the fiscal crisis in two ways. First, the implementation timeline was "moved back" in time, to allow institutions to begin to recover, if possible, from recent budget reductions. Second, the plan offered two ways for institutions to respond to the standards, depending on local fiscal conditions. On October 2, 1992, the Commission adopted this implementation plan, which appears on the following page. The implementation timeline is summarized on page 47, and diagrammed on 48.

Implementation Timeline: Impact on Prospective Mathematics Teachers

Based on the Commission's implementation plan, candidates for Single Subject Credentials in Mathematics who do not plan to pass the Commission-adopted subject matter examinations should enroll in subject matter programs that fulfill the "new" standards either (1) once a new program commences at their institution or (2) by January 1, 1995, whichever occurs first. After a new program begins at an institution, no students should enroll for the first time in an "old" program (i.e. one approved under "old" guidelines). Regardless of the date when new programs are implemented, no students should enter old programs after January 1, 1995.

Candidates who enrolled in programs approved on the basis of pre-1991 guidelines ("old" programs) may complete those programs provided that (1) they entered the old programs either before new programs were available at their institutions, or before January 1, 1995, and (2) they complete the old programs before January 1, 1998.

Candidates who do not comply with these timelines may qualify for Single Subject Teaching Credentials by passing the subject matter examinations that have been adopted for that purpose by the Commission.

Implementation Plan Adopted by the Commission

October 2, 1992

- (1) The Commission will review two kinds of proposals that respond to the Standards of Program Quality in Mathematics. The Commission will grant *full approval* to programs that satisfy the full complement of standards in this handbook, based on the judgments of the program reviewers. The Commission will grant *interim approval* to programs that satisfy the full complement except for one or more of the standards concerning Program Coordination (Standard 18), Student Advisement (19), and Student Assessment (20).
- (2) An institution may seek full approval of some programs and interim approval of other programs. To seek full approval of a program, an institution must respond to all of the standards. To seek interim approval of a mathematics program, the institution must respond to all of the standards except Standards 18, 19 and 20.
- (3) By January 1, 1995, existing ("old") programs based on current guidelines should be superseded by new programs with either full approval or interim approval.
 - (a) Once a new program receives full or interim approval, all students not previously enrolled in the old program (i.e., all "new" students) should enroll in the new program.
 - (b) After January 1, 1995, no "new" students should enroll in an "old" program, even if a new program in the subject is not available at that institution.
 - (c) Students who enrolled in an old program prior to January 1, 1995, may continue to pursue the old program [see (5) below].
- (4) By January 1, 1998, a program with interim approval must earn full approval. To seek full approval of a mathematics program with interim approval, the institution should respond only to Standards 18, 19 and 20. If the program satisfies these standards, the Commission will grant full approval. An institution may seek full approval of a program with interim approval any time between the granting of interim approval and January 1, 1998.
- (5) Until January 1, 1998, students may qualify for examination waivers based on "old" program guidelines provided that the students entered the old program prior to either (a) the implementation of a new program with full approval or interim approval at their institution, or (b) January 1, 1995, whichever occurs first.

Timeline for Implementing the Mathematics Standards

November 1991	The Commission on Teacher Credentialing adopts the Standards of Program Quality and Effectiveness that are on pages 15-36 of this handbook, as well as the preconditions on page 14.
October 1992	The Commission adopts the plan, on page 46 of the handbook, for implementing the standards and preconditions.
January to March, 1993	The Executive Director disseminates the handbook. The Commission conducts regional workshops to answer questions, provide information, and assist colleges and universities.
May to August, 1993	The Commission selects, orients and trains a Program Review Panel in Mathematics. Qualified subject matter experts are prepared to review programs in relation to the standards beginning in 1993-94.
September 1993	Review and approval of programs under the new standards begins. No new subject matter programs in mathematics will be reviewed in relation to the Commission's "old" guidelines.
1993-94 1994-95	Institutions may submit programs for preliminary or formal review on or after September 1, 1993. Once a "new" program is approved, all students who were not previously enrolled in the "old" program (i.e., all new students) should enroll in the new program. Students may complete an old program if they enrolled in it either (1) prior to the commencement of the new program at their campus, or (2) prior to January 1, 1995, whichever occurs first.
Jan. 1, 1995	"Old" programs that are based on pre-1991 guidelines must be superseded by new programs with either full approval or interim approval (see pages 52-53). After January 1, 1995, no new students should enroll in an old program, even if a new program in mathematics is not yet available at the institution.
1995-96 1996-97 1997-98	The Commission will continue to review program proposals based on the standards and preconditions in this handbook. Institutions with interim approval of a program may seek full approval of that program at any time before January 1, 1998.
January 1, 1998	A program with interim approval must earn full approval by the Commission. To seek full approval of a program with interim approval, the institution should respond to Standards 18, 19, and 20.
January 1, 1998	The final date for candidates to complete subject matter preparation programs approved under the pre-1991 guidelines. To qualify for a credential based on an "old" program, students must have entered that program prior to either (1) the implementation of a new program with full or interim approval at their institution, or (2) January 1, 1995, whichever occurs first.

Implementation Timeline Diagram

November 1991

Adopt the mathematics standards and preconditions in this handbook.

October 1992

Adopt the revised timeline and implementation plan for the mathematics standards.

January to March, 1993

Disseminate the standards, timeline and implementation plan throughout the state. Hold regional workshops to offer information, answer questions, and assist colleges and universities.

September 1993

Colleges and universities may begin to present program documents for review by the Commission's staff and Program Review Panels.

January 1, 1995

"Old" subject matter programs in mathematics must be superceded by new programs with full approval or interim approval.

January 1, 1998

A program with interim approval must earn full approval by the Commission.

January 1, 1998

Final date for candidates to qualify for Single Subject Credentials in Mathematics on the basis of "old" programs of subject matter preparation.

Implementation Handbook: Review and Approval of Subject Matter Programs in Mathematics

A regionally accredited institution of postsecondary education that would like to offer (or continue to offer) a Program of Subject Matter Preparation for the Single Subject Credential in Mathematics may present a program proposal that responds to the standards and preconditions in this handbook. The submission of programs for review and approval is voluntary for colleges and universities; candidates can qualify for the Single Subject Credential by passing a standardized assessment of their knowledge and competence in mathematics.

For a subject matter program in mathematics to be approved by the Commission, it must satisfy the preconditions and standards in this handbook. If an institution would like to offer two or more distinct programs of subject matter preparation in mathematics, a separate proposal should be forwarded to the Commission for each program. For example, one program in mathematics might have an emphasis in computer science, while a second program at the same institution could have an emphasis in mathematical applications or critical thinking.

The Commission is prepared to review subject matter program proposals beginning on September 1, 1993. Prior to that date, the Commission's professional staff is able to consult with institutional representatives, and to do preliminary reviews of draft proposals (see page 50 for details).

Initial Statement of Institutional Intent

To assist the Commission in planning and scheduling reviews of program proposals, each institution is asked to file a Statement of Intent at least four months prior to submitting a proposal. Having received a timely Statement of Intent, the Commission will make every effort to review a proposal expeditiously. In the absence of a timely statement, the review process will take longer.

The Statement of Intent should be signed by the individual with chief responsibility for academic programs at the institution. It should provide the following information:

- The subject for which approval is being requested (mathematics).
- The contact person responsible for each program (include phone number).
- The expected date when students would initially "enroll" in each program.
- An indication as to whether or not the institution expects to submit a program for "informal" review (defined below).
- The date when each program will be submitted for formal review and approval.

If an institution plans to submit proposals for two or more programs in mathematics, the Statement of Intent should include this essential information for each program, and should indicate whether or not the programs will have distinct emphases.

The Program Document or Proposal

For each program, the institution should prepare a program document that includes a narrative response to each precondition and standard on pages 14-36. Please provide six (6) copies of each program document.

Preconditions. A narrative section of the document should explain how the program will meet each precondition on page 14. In responding to the preconditions, the document must show the title and unit value of each required and elective course in the basic core of the program (Precondition 2) and the breadth and perspective component (Precondition 3). The document must also include brief course descriptions.

Standards. In the major part of the program document, the institution should respond to each Standard of Program Quality and Effectiveness on pages 15-36. It is important to respond to each element of a standard, but a lengthy, detailed description is not necessary. An institution's program document should include syllabi of required and elective courses, to serve as "back-up" information for responses to particular standards.

Factors to Consider. A program proposal must show how the program will meet each standard. The purpose of factors to consider is to amplify specific aspects of standards, and to assist institutions in responding to all elements of a standard. The Commission considers the factors to be important aspects of program quality, but it is not essential that the document respond to every factor. The factors are *not* "mini-standards," and there is *no expectation* that a program must "meet" all the factors in order to fulfill a standard. (For added information about factors to consider, see pages 4 and 13.)

Institutions are urged to *reflect on* the factors to consider, which may or may not be used as the "organizers" or "headings" for an institution's response to a standard. The quality of a program may be enhanced by an "additional factor" that is related to a standard but not represented by any of the adopted factors. Institutions are encouraged to describe all aspects of the program's quality, and not limit their responses to the adopted factors in this handbook.

Steps in the Review of Programs

The Commission is committed to conducting a program review process that is objective, authoritative and comprehensive. The agency also seeks to be as helpful as possible to colleges and universities throughout the review process.

Preliminary Staff Review. Before submitting program documents for formal review and approval, institutions are encouraged to request preliminary reviews of *draft* documents by the Commission's professional staff. The purpose of these reviews is to assist institutions in developing programs that are consistent with the intent and scope of the standards, and that will be logical and clear to the external reviewers. Program documents may be submitted for preliminary staff review at any time; the optimum time is at least one month after submitting the Statement of Intent and at least two months prior to the expected date for submitting a completed document. Preliminary review is voluntary; its purpose is to assist institutions in preparing program documents that can be reviewed most expeditiously in the formal review process.

Implementation Handbook: Review and Approval of Programs in Mathematics

Review of Preconditions. An institution's response to the preconditions is reviewed by the Commission's professional staff because the preconditions are based on state laws and regulations, and do not involve issues of program quality. At the institution's discretion, preconditions may be reviewed either during the preliminary review stage, or after the institution's formal submission of a document. If the staff determines that the program complies with the requirements of state laws and administrative regulations, the program is eligible for a review of the standards by a panel of subject matter experts. If the program does not comply with the preconditions, the staff returns the document to the institution with specific information about the lack of compliance. Such a program may be resubmitted once the compliance issues have been resolved.

Review of Program Quality Standards. Unlike the preconditions, the standards address issues of program quality and effectiveness, so each institution's response to the standards is reviewed by a small Program Review Panel of subject matter experts. During the review process, there is an opportunity for institutional representatives to meet with the Program Review Panel to answer questions or clarify issues that may arise. Prior to such a discussion, the panel will be asked to provide a preliminary written statement of the questions, issues or concerns to be discussed with the college or university representative(s).

If the Program Review Panel determines that a proposed program fulfills the standards, the Commission's staff recommends the program for approval by the Commission during a public meeting no more than eight weeks after the panel's decision.

If the Program Review Panel determines that the program does not meet the standards, the document is returned to the institution with an explanation of the panel's findings. Specific reasons for the panel's decision are communicated to the institution. If the panel has substantive concerns about one or more aspects of program quality, representatives of the institution can obtain information and assistance from the Commission's staff. With the staff's prior authorization, the college or university may also obtain information and assistance from one or more designated members of the panel. After changes have been made in the program, the proposal may be re-submitted to the Commission's staff for re-consideration by the panel.

If the Program Review Panel determines that minor or technical changes should be made in a program, the responsibility for reviewing the re-submitted document rests with the Commission's professional staff, which presents the *revised* program to the Commission for approval without further review by the panel.

Appeal of an Adverse Decision. An institution that would like to appeal a decision of the staff (regarding preconditions) or the Program Review Panel (regarding standards) may do so by submitting the appeal to the Executive Director of the Commission. The institution should include the following information in the appeal:

- The original program document, and the stated reasons of the Commission's staff or the review panel for not recommending approval of the program.
- A specific response by the institution to the initial denial, including a copy of the resubmitted document (if it has been resubmitted).
- A rationale for the appeal by the institution.

The Executive Director may deny the appeal, or appoint an independent review panel, or present the appeal directly to the Commission for consideration.

Responses to Six Common Standards

The Commission adopted six of the standards for programs in *all* single subject disciplines.

- Standard 1. Program Philosophy and Purpose.
- Standard 15. Equity and Diversity in the Program.
- Standard 18. Coordination of the Program.
- Standard 19. Student Advisement and Support.
- Standard 20. Assessment of Subject Matter Competence.
- Standard 21. Program Review and Development.

These six standards are referred to as “common standards” because they are essentially the same in all subject areas.

An institution’s program document in mathematics should include a subject-specific response to Standards 1 and 15, along with subject-specific responses to the other curriculum standards in Category I (see pp. 15-32). An institution’s program document in mathematics *may* also include a unique response to Standards 18, 19, 20 and 21. Alternatively, the institution *may* submit a “generic response” to these four common standards. In a generic response, the institution should describe how credential preparation programs in all subjects will meet the four standards. A generic response should include sufficient information to enable an interdisciplinary panel of reviewers to determine that the four common standards are met in each subject area. Once the institution’s generic response is approved, it would not be necessary to respond to the four standards in the institution’s program document in mathematics, or in any other subject. (Institutions seeking “interim approval” may submit a generic response to Standard 21 only. See below for information about interim approval.)

Full Approval and Interim Approval

Even after the Commission adopted the standards in this document, Commissioners were concerned that *some of the standards* might be prohibitively expensive for some institutions to implement during the current fiscal crisis. At the same time, the Commission did not want to delay implementation of all the standards by those institutions that can do so in the near term. To accommodate differences among institutions, the Commission created two options: either address all of the standards or address all except 18, 19 and 20.

If the Program Review Panel determines that a program fulfills all of the standards, the panel will recommend *full approval* of the program by the Commission. If the panel finds that a program satisfies all of the standards except Standards 18, 19, and 20, it will recommend that the Commission grant *interim approval* to the program. The latter option will be available from 1993-94 through 1996-97.

To seek *full approval* of a program, the institution must address all standards. To seek *interim approval*, the initial program document must address all standards except 18, 19 and 20. If the document addresses all standards, and the Review Panel finds that all standards are met except 18-20, the Commission’s staff consultant will contact the institution to determine if the Commission should grant interim approval to the program. The alternative in this case would be for the institution to re-submit the proposal for full approval after revising it in relation to Standards 18, 19, and/or 20.

Programs with interim approval must earn full approval before January 1, 1998. An institution that sponsors programs with interim approval may seek full approval at any time during 1993-94, 1994-95, 1995-96 or 1996-97. To seek full approval, the institution needs to respond only to standards that were not addressed in the initial program document. If the Review Panel determines that these standards are met, the panel will recommend that the Commission grant full approval to the program.

Selection, Composition and Training of Program Review Panels

Review panel members are selected because of their expertise in mathematics, and their knowledge of mathematics curriculum and instruction in the public schools of California. Reviewers are selected from institutions of higher education, school districts, county offices of education, organizations of subject matter experts, and statewide professional organizations. Members are selected according to the Commission's adopted policies that govern the selection of panels. Members of the Commission's former Single Subject Waiver Panels and Subject Matter Advisory Panels may be selected to serve on Program Review Panels.

The Program Review Panel in Mathematics includes at least one professor of mathematics, at least one high school teacher of mathematics, and a third member who is either another professor, another teacher, or a curriculum specialist in mathematics.

The Program Review Panel is trained by the Commission's staff. Training includes:

- The purpose and function of subject matter preparation programs.
- The Commission's legal responsibilities in program review and approval.
- The role of the review panel in making program determinations.
- The role of the Commission's professional staff in assisting the panel.
- A thorough analysis and discussion of each standard and rationale.
- Alternative ways in which the standard could be met.
- An overview of review panel procedures.
- Simulated practice in reviewing programs.
- How to write program review panel reports.

The initial phase of training involves panels that have been selected to review programs in several subject areas, and includes training in the Common Standards. In the concluding phase, the reviewers of mathematics programs are trained specifically in the consistent application of the subject-specific standards in mathematics.

Program Review Panel Procedures

The Program Review Panel meets periodically to review programs that have been submitted to the Commission during a given time period. Review meetings usually take place over three days, and typically adhere to the following general schedule:

- First Day - Review institutional responses to common standards. Preliminary discussion of responses to curriculum standards.
- Second Day - Thorough analysis of responses to curriculum standards. Prepare preliminary written findings for each program, and FAX these to institutions.

Implementation Handbook: Review and Approval of Programs in Mathematics

- Third Day - Meet with representatives of institutions to clarify program information, discuss preliminary findings and identify possible changes in programs. Prepare written reports that reflect the discussions with institutions.

Normally, the Program Review Panel's written report is mailed to the institution within two weeks after the panel meeting. If the report is affirmative, the Commission's staff presents the report to the Commission during a public meeting no more than eight weeks after the panel's decision.

If the report indicates that the program does not meet the standards, specific reasons for the panel's decision are included in the report. The institution should first discuss the report with the Commission's staff. One or more designated members of the panel may also be contacted, but only after such contacts are authorized by the staff.

If the report shows that minor or technical changes are needed in a program, the review panel gives responsibility for reviewing the re-submitted document to the staff.

Whenever possible, Program Review Panels in more than one subject meet at the same time and location. This enables institutional representatives to meet with reviewers in more than one subject area, if necessary. It also facilitates reviews of the common standards, and utilizes the Commission's staff resources most efficiently.

Further Information and Communications Related to Standards, Programs and Program Reviews

Regional Workshops for Colleges and Universities

During March, 1993, the Commission will sponsor three regional workshops to provide assistance to institutions related to their subject matter programs in mathematics. The agenda for each workshop will include:

- Explanation of the implementation plan adopted by the Commission.
- Description of the steps in program review and approval.
- Review of program standards, factors to consider, preconditions, and examples presented by Subject Matter Advisory Panel members and others with experience in implementing Standards of Program Quality.
- Opportunities to discuss subject-specific questions in small groups.

All institutions that plan to submit program documents (or are considering this option) are welcome to participate in the workshops. Specific information about the workshop dates and locations is provided separately from this handbook.

Communications with the Commission's Staff and Program Review Panel

The Commission would like the program review process to be as helpful as possible to colleges and universities. Because a large number of institutions prepare teachers in California, representatives of an institution should first consult with the Commission's professional staff regarding programs that are in preparation or under review. The staff responds to all inquiries expeditiously and knowledgeably. Representatives of colleges and universities should contact members of a Program Review Panel only when they are authorized to do so by the Commission's staff. This restriction must be observed to ensure that membership on a panel is manageable for the reviewers. If an institution finds that needed information is not sufficiently available, please inform the designated staff consultant. If the problem is not corrected in a timely way, please contact the Executive Director of the Commission.

Request for Assistance from Handbook Users

The Commission welcomes comments about this handbook, which should be addressed to:

Commission on Teacher Credentialing
Professional Services Division
1812 Ninth Street
Sacramento, California 95814-7000